

Chapter 10 / **Example 19****Finding roots of a complex number**

Use your calculator to find all the solutions of the equation $z^5 = 2 + 5i$ in Cartesian form.

Find the fifth root of $2 + 5i$ and store it as Z.

To enter $\sqrt[5]{2 + 5i}$ type $(2 + 5i)^{\frac{1}{5}}$, press **[STO▶]** **[ALPHA]** Z and press **[ENTER]**.

Use the fraction template by pressing **[ALPHA]** **[f1]** 1:n/d.

```
(2+5i)^(1/5)→Z
.....1.360867038+.3302271356i
```

Where $z = r \operatorname{cis} \theta$, $\sqrt[n]{z} = \sqrt[n]{r} \operatorname{cis} \frac{\theta + 2k\pi}{n}$, $k = 0, 1, 2, \dots, n - 1$.

Find r and store it by typing $|Z|$ **[STO▶]** **[ALPHA]** R and press **[ENTER]**.

To enter the modulus function press **[math]** ▶ NUM 1:abs(.

To enter i press **[2nd]** **[.]** **[i]**.

```
(2+5i)^(1/5)→Z
.....1.360867038+.3302271356i
|Z|→R
.....1.400360331
```

Find θ and store it. Press **[math]** ▶▶ CMPLX 4:angle(, type **[ALPHA]** Z, close the parentheses, type **[STO▶]** **[ALPHA]** θ and press **[ENTER]**.

```
(2+5i)^(1/5)→Z
.....1.360867038+.3302271356i
|Z|→R
.....1.400360331
angle(Z)→θ
......2380579899
```

Now use the formula for arguments to find all the solutions and convert these to Cartesian form.

Type $r \times e^{\left(\theta + \frac{2\pi}{5}\right)i}$.

Since the default format of complex numbers is Cartesian, the GDC converts from polar to Cartesian format without needing to enter any additional command.

```
R×e^(θ+2π/5)i
.....1.064663728+1.396307262i
```

Chapter 10 / **Example 19**

Finding roots of a complex number

To repeat this calculation, select it and press **ENTER**.

Edit the expression, changing 2 to 4 and press **ENTER**.

$$\begin{aligned} &R\angle e^{i\left(0+\frac{2\pi}{5}\right)} \\ &\dots .1064663728+1.396307262i \\ &R\angle e^{i\left(0+\frac{4\pi}{5}\right)} \\ &\dots -1.295067201+.5327382109i \end{aligned}$$

Repeat changing 4 to 6 and press **ENTER**.

$$\begin{aligned} &R\angle e^{i\left(0+\frac{2\pi}{5}\right)} \\ &\dots .1064663728+1.396307262i \\ &R\angle e^{i\left(0+\frac{4\pi}{5}\right)} \\ &\dots -1.295067201+.5327382109i \\ &R\angle e^{i\left(0+\frac{6\pi}{5}\right)} \\ &\dots -.906861921-1.06705694i \end{aligned}$$

Repeat changing 6 to 8 and press **ENTER**.

$$\begin{aligned} z_1 &= 1.36 + 0.330i, z_2 = 0.106 + 1.40i, z_3 = -1.30 + 0.533i, \\ z_4 &= -0.907 - 1.07i, z_5 = 0.735 - 1.19i. \end{aligned}$$

$$\begin{aligned} &R\angle e^{i\left(0+\frac{2\pi}{5}\right)} \\ &\dots -1.295067201+.5327382109i \\ &R\angle e^{i\left(0+\frac{6\pi}{5}\right)} \\ &\dots -.906861921-1.06705694i \\ &R\angle e^{i\left(0+\frac{8\pi}{5}\right)} \\ &\dots .7345957111-1.192215668i \end{aligned}$$